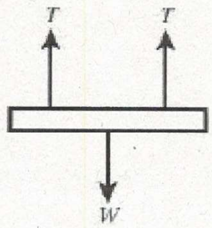
Forces and Equilibrium Worksheet Solutions

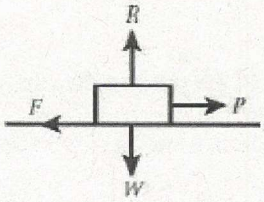
1.

T is the tension in each of the ropes.
W is the weight of the bar.



2.

R is the normal reaction of the road on the car.
W is the weight of the car.
F is the sum of the frictional forces on the car.
P is the forward force produced by the car's engine.



3.

Since the particle is at rest, both horizontal and vertical forces must be balanced.
Considering horizontal forces only:
$p - 50 = 0$
$p = 50$
Considering vertical forces only:
$5q - (q + 10) - 3p = 0$
$4q - 10 - (3 \times 50) = 0$
$4q = 160$
$q = 40$
The values of p and q are 50 and 40 respectively.

4.

Since the particle is moving with constant velocity, both horizontal and vertical forces must be balanced.
Considering horizontal forces only:
$2P + Q = 25$
$Q = 25 - 2P$
Considering vertical forces only:
$3P - 2Q = 20$
Substituting for Q:
$3P - 2 \times (25 - 2P) = 20$
$3P - 50 + 4P = 20$
$7P = 20 + 50 = 70$
$P = 10\text{ N}$
Using this value of P in the horizontal equation:
$(2 \times 10) + Q = 25$
$Q = 25 - 20 = 5$
$Q = 5\text{ N}$
P is 10 N and Q is 5 N.

5.

a **i** Overall horizontal force = 100 – 100 = 0
Overall vertical force = 40 – 20 = 20
The resultant force is 20 N upward.

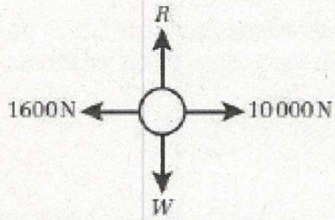
ii The particle accelerates vertically upward.

b **i** Overall horizontal force = 25 – 5 = 20
Overall vertical force = 10 – 10 = 0
The resultant force is 20 N to the right.

ii The particle accelerates to the right.

6.

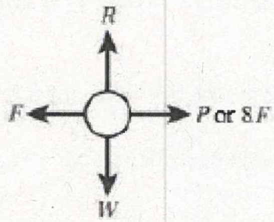
a R is the normal reaction of the road on the car.
W is the weight of the car.
The forward thrust of the car's engine acts to the right in the diagram.
The car is travelling to the right (positive direction).
The frictional forces on the car are acting to the left.



b Considering horizontal forces only:
Resultant force = 10 000 – 1600
There is no overall vertical force: R and W must be balanced, otherwise the car would lift off the road or sink into it.
The resultant force is 8400 N in the direction of travel.

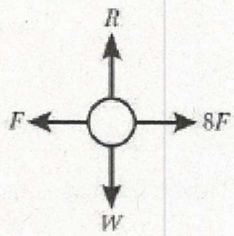
7.

a R is the normal reaction of the road on the car.
W is the weight of the car.
P is the driving force produced by the car's engine.
F is the resistance to the car's motion. or



b The magnitude of the driving force is eight times the magnitude of the resistance force, so

$P = 8F$



The resultant force is the difference between the forward force P and the resistance force F, so

$8F - F = 7F = 4200$

$$F = \frac{4200}{7} = 600$$

The magnitude of the resistance force is 600 N.

8.

Since object is in equilibrium:

$$\begin{pmatrix} a \\ 2b \end{pmatrix} + \begin{pmatrix} -2a \\ -b \end{pmatrix} + \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a \\ 2b \end{pmatrix} + \begin{pmatrix} -2a \\ -b \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} -a \\ b \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

$a = 3$ and $b = 4$

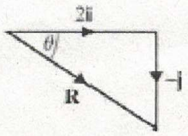
9.

a **i** $(-2\mathbf{i} + \mathbf{j}) + (5\mathbf{i} + 2\mathbf{j}) + (-\mathbf{i} - 4\mathbf{j}) = (2\mathbf{i} - \mathbf{j})$

The resultant vector is $(2\mathbf{i} - \mathbf{j})$ N.

ii $\sqrt{2^2 + 1^2} = \sqrt{5}$

The magnitude of the resultant vector is $\sqrt{5}$ N.



iii $\tan\theta = \frac{1}{2}$

$\theta = -26.6°$ This is the angle made from east, with **anticlockwise** defined as positive.

The **bearing** is the angle made from **north**, with **clockwise** defined as positive $= 90 - \theta$

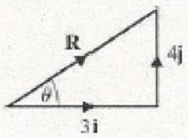
The force acts at a bearing of 116.6°.

b **i** $(-2\mathbf{i} + \mathbf{j}) + (2\mathbf{i} - 3\mathbf{j}) + (3\mathbf{i} + 6\mathbf{j}) = (3\mathbf{i} + 4\mathbf{j})$

The resultant vector is $(3\mathbf{i} + 4\mathbf{j})$ N

ii $\sqrt{3^2 + 4^2} = \sqrt{25}$

The resultant force is 5 N.



iii $\tan\theta = \frac{4}{3}$

$\theta = 53.1°$ This is the angle made from east, with **anticlockwise** defined as positive.

The **bearing** is the angle made from **north**, with **clockwise** defined as positive $= 90 - \theta$

The force acts at a bearing of 036.9°.

10.

Since the object is in equilibrium:
$(2a\mathbf{i} + 2b\mathbf{j}) + (-5b\mathbf{i} + 3a\mathbf{j}) + (-11\mathbf{i} - 7\mathbf{j}) = 0$

Considering **i** components:
$2a - 5b - 11 = 0$ **(1)**

Considering **j** components:
$2b + 3a - 7 = 0$ **(2)**

equation (1) × 3 → $6a - 15b - 33 = 0$ (3)
equation (2) × 2 → $6a + 4b - 14 = 0$ (4)

Subtracting (4) from (3):
$-15b - 33 - 4b - (-14) = 0$
$-19b = 33 - 14$
$b = -1$
Substituting this value into equation (1):
$2a - 5(-1) - 11 = 0$
$2a = 11 - 5 = 6$
The values of a and b are 3 and −1, respectively.

11.

a Since the particle P is in equilibrium:

$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$$\begin{pmatrix}-7\\-4\end{pmatrix} + \begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}7\\4\end{pmatrix} + \begin{pmatrix}-4\\-2\end{pmatrix}$$

$$\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}3\\2\end{pmatrix}$$

The values are $a = 3$, $b = 2$

b $\mathbf{R} = \mathbf{F}_2 + \mathbf{F}_3$

$$\mathbf{R} = \begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}7\\4\end{pmatrix}$$